SET ALGEBRAIC REPRESENTATION OF SOME ASPECTS OF THE ORGANON

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ABSTRACT: Hahnemann visualised, "The healing art will then come near the mathematical science in certainty." Here it is a modest attempt to make his vision come true. In this paper the disease symptoms, generals and particulars, common and characteristics, have been assumed to be the elements of a set of disease symptoms S(D). The symptoms of a medicine as described in the materia medica have been taken as the elements of a set of remedy symptoms S(R). A time-dependent annihilator operator $A^+(t)$ has been assumed to operate on S(R) $\cap$ S(D) when the remedy R has been administered to the patient, and a set algebraic representation has been derived for the cure state. Set theoretic representation of a few sections of the Organon have been proposed, and it has been shown by appropriate references that a single remedy cure is only a particular case of treatment homoeopathically and polyprescription treatment, i.e. prescribing more than one remedy at a time is the general case.

Let S(D) represent the set of symptoms as described by the patient, by the person or persons attending the patient, as observed by the physician (objective symptoms), any relevant rubrics collected by the physician from the patient, past history, family history of the patient etc. Let these be represented by $S_1(D)$, $S_2(D)$, $S_3(D)$...............$S_n(D)$. Then these n number of symptoms are the number of elements of the set S(D), so that

$$S(D) = \{ S_1(D), S_2(D), S_3(D), ... \}$$

These symptom elements are time dependent, and may be continuous or periodic function of time. An untreated benign or malignant growth in the body is an example of a symptom which is a continuously increasing function time; the variation may be linear usually in case of benign, and exponential in case of malignant growth. Yearly winter skin symptom of Psorinum, some symptoms of China, Cedron, etc. are periodic function of time. Similarly we write:

$$S(R) = \{ S_1(R), S_2(R), S_3(R)................ S_n(R) \}$$

Where S(R) represent the set of remedy symptoms. $S_1(R)$, $S_2(R)$......etc. being the elements of the set as described in the materia medica. In treating the case homoeopathically, the physician selects a remedy R such that

$$S_1(R), S_2(R), S_3(R)................S_n(R)$$

are exactly similar, as regards location, sensation, modality and concomitants, to the elements of $S_1(D)$, $S_2(D)$, $S_3(D)$...........$S_n(D)$. When such a selection has been successfully made, then the remedy picture will exactly fit into the disease picture, and there will be an 1 to 1 correspondence of the disease and remedy (medicinal) symptoms. When an 1 to 1 correspondence exists between the element of S(D) and S(R), then one may write by property of set algebra,

$$S(R) \subseteq S(D), \text{ and } S(D) \subseteq S(R),$$

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where $\subseteq$ is read as 'contained'. The first representation indicates that the elements of remedy symptoms are contained in the set S(D); and the second representation indicates that elements of the disease symptoms are contained in the set S(R). When above conditions have been achieved, one may write:

$$S(D) = S(R), \text{ i.e. the two}$$

sets are identical or similar, and only under such condition one may claim that totality of symptoms of the disease and the remedy has been attained and then only single remedy will cure.

But it very often happens that a homoeopathic physician finds that all the disease symptoms are not contained in a single remedy, i.e.

$$S(R) \subseteq S(D) \text{ and } S(D) \subseteq S(R),$$

are not satisfied. Hahnemann himself had noted it and he described it as, "It sometimes happens, *owing to the moderate number of medicines yet known with respect to true, pure action,* that but a *portion* of the symptoms of the disease under treatment are to be met with in the list of symptoms of the most appropriate medicine, consequently thus imperfect medicinal morbific agent must be employed for lack of a more perfect one.—*Organon,* 5th ed., sec. 162. Expressed mathematically by set algebra one may write:

$$S(R) \subseteq S(D), \text{ but } S(D) \not\subseteq S(R)$$

The physical meaning of these relations is that the set of remedy symptoms is contained in the elements of the disease symptoms, but the set of disease symptoms is not contained in the set of remedy symptoms S(R). Mathematically this is expressed by saying that S(R) is a subset of S(D), physically meaning thereby that S(D) has some elements different from S(R). Under such condition one may write:

$$S(D) \neq S(R).$$

Let us now see what Hahnemann said about the consequence of administration of imperfect medicinal morbific agent for the lack of a more perfect one. He said, "In this case we cannot indeed expect from this medicine a complete untroubled cure; for during its use some symptoms appear which were not previously observable in the disease, accessory symptoms of the not perfectly appropriate remedy. This does by no means prevent a considerable part of the disease (the symptoms of the disease that resemble those of the medicine) from being eradicated by this medicine, thereby establishing a fair commencement of the cure, but still this does not take place without those accessory symptoms, which are, however, always moderate when the dose of the medicine is sufficiently minute."—*Organon,* 5th ed., sec. 163.

Let us describe an illustration to explain the inner meaning of the section.

A young girl aged 17, had her first serious impairment of health at her puberty, which was delayed. She has "never been well since". Bad taste in

the mouth, with dry tongue and no thirst. She is anaemic and has recurrent attacks of bronchitis; usually constipated since her first menstruation, but has frequent attacks of diarrhoea, usually at night. She weeps easily, indecisive, slow and of phlegmatic temperament, desires warm food, good appetite, but a few mouthfuls fill up to the throat. She has right sided overalgia which changes side during menstruation. All her symptoms are < from 4 to 8 p.m., especially excessive accumulation of flatus, sour eructation, heartburn and headache.

To a homoeopathic physician the disease-picture will indicate two medicines, (i) Lycopodium, (ii) Pulsatilla. Since a medicine cannot cure which it does not produce, it is evident that to achieve a cure both the remedies have to be administered. Selection of a single remedy here will correspond to an application of imperfect medicinal morbific agent. Hence the result will be only a partial cure, not full. Let R_1 represent Lycopodium and R_2 Pulsatilla, then

$$S(R_1) \subseteq S(D) \text{ and } S(R_2) \subseteq S(D)$$

i.e. some symptoms of R_1 and R_2 are subsets of the set of symptoms of the disease. When first R_1 is administered we have a partial similimum of the elements of the disease symptoms with those of Lycopodium. This we mathematically expressed as

$$S(R_1) \cap S(D) \text{ and } S(R_2) \cap S(D).$$

when R_2, i.e. Pulsatilla is administered. Here $\cap$ is read as intersection of the two sets, its mathematical interpretation is that a new set is formed by the elements common to both $S(R_1)$ and $S(D)$. Let us represent the remaining symptoms of $S(D)$, which do not have an 1 to 1 correspondence with the elements of $S(R_1)$ by $S^*(D)$. It is then evident that in this case, for cure, one must have

$$S^*(D) \subseteq S(R_2) \text{ and } S(R_2) \subseteq S^*(D).$$

But in section 163 Hahnemann has expressed the idea that the imperfectly selected remedy will develop some accessory symptoms not previously observable in the disease. Let us represent the set of accessory symptoms by S(A). Now, in section 29 Hahnemann has said that the medicine will annihilate the disease symptoms which are similar to its own. We express this physical phenomenon mathematically by the introduction of an operator $A_1^+(t)$, the annihilator operator which is time dependent, and its nature is such that it will only act when there is a superimposition of $S(R_1)$ and $S(D)$ through the application of the remedy R_1; and when it acts it reduces the magnitude of the disease elements common to the both sets and finally makes them zero. This is physically interpreted as the amelioration of the disease symptoms, i.e. the beginning of the cure process. The partially cured state of the patient is represented mathematically as

$$A^+(t) \, [\, S(R_1) \cap S(D) \,] = S^*(D) \cup S(A).$$

where S(A) is the set of accessory symptoms. U is read as union of two sets S(D) and S(A). By the property of union of two sets we know that a new set is formed whose elements are composed of the accessory symptoms and the disease symptoms not cured by the remedy R₁. S(A) is formed by the action of R₁ on the system, i.e. the patient.

It sometimes so happens that the administration of the remedy is followed by an aggravation of the disease symptoms which Hahnemann defined as the "homoeopathic aggravation". This phenomenon may be mathematically expressed by introduction of a creator operator, $C_1(t)$, which is time-dependent, and related to the remedy. R_1. $C_1(t)$ operating on the elements of the set $S(R_1) \cap S(D)$, enhances their magnitude. We express this as:

$$C_1(t) \, [\, S(R_1) \cap S(D) \,] = K \, [\, S(R_1) \cap S(D) \,]$$

Aggravation is followed by gradual amelioration of the disease symptoms when the annihilation operator becomes operative. Therefore,

$$A_1^+(t) \, [\, C_1(t) \, \{ S(R_1) \cap S(D) \} \,] = S^*(D) \cup S(A)$$

Physically this equation means that now $A_1^+(t)$ acts on the aggravated symptoms elements of $S(R_1) \cap S(D)$.

Let $\Psi_{R_1}(P,t)$ represent the action of the R_1 remedy in the system. It is a decreasing function of time, and potency, i.e.

$$\left[\frac{d}{dt} \, \Psi_{R_1}(P,t) \right]_{t=t_1} = 0$$

also $\left[\Psi_{R_1}(P,t) \right]_{t=t_1} = 0$, and when $\Psi_{R_1}(P,t) = 0$, $A_1^+(t) = 0$

This means that curative action of the remedy is exhausted after time t_1, from the administration of the remedy R_1 to the patient. This time t_1 corresponds to the "duration of action" of the remedy as described by Hahnemann. The total medicinal effect of the remedy R_1 is represented mathematically as

$$M_1 = \int_0^{t_1} \Psi_{R_1}(P,t) \, dt.$$

Higher the potency, smaller is the duration of action of the remedy[3]. Also higher the potency smaller is the medicinal effect of the remedy[4]. Smaller the dose, smaller is the value of the medicinal effect of the remedy. Effective dose varies directly as its volume[5]. From; (3—5), volume of a dose remaining constant, it is evident that $\Psi_{R_1}(P,t)$ varies inversely as some function of potency of the remedy, say F(P). One may conclude from the analysis above that a dose of CM potency of a remedy will have shorter duration of action, smaller medicinal effect and less aggravation than a 10M potency dose of the same remedy. Hence higher potency remedies admit more frequent

repetition of the remedy. Following the method of divided doses as described by Hahnemann, the author repeats the indicated remedy or remedies even in CM potencies several times a day in both acute and chronic cases, resulting in cure very rapidly.

The elements of S(A) may be (i) the symptoms which the patient had suffered previously, but not present now, and belonging to the set S(R$_2$), i.e. in the illustration taken above, some symptoms of Pulsatilla, (ii) reappearance of the past symptoms, but not belonging to S(R$_2$), (iii) drug proving symptoms of the remedy R$_1$. It is evident that if the accessory symptoms belong to R$_2$, then

$$A_2^+(t) [S(R_2) \cap (S^*(D) \cup S(A))] = 0$$

This indicates that total elimination of the symptoms takes place after the administration of the remedies R$_1$ and R$_2$. If S(A) contains only the drug symptoms of R$_1$, then also above equation will be valid, because the vital force will itself eliminate without help of any medicine, the drug symptoms of R$_1$, and thus the elements of S(A). If however, the elements of S(A) belong to some other remedy R$_3$, then two possibilities may arise: (i) R$_3$ will eliminate the symptoms of S*(D); and vital force being empowered by the action of the remedies R$_1$ and R$_3$ will itself eliminate the symptoms of S(A)', (ii) a remedy R$_3$ has to be selected so that S(R$_3$) = S(A).

In section 169, Hahnemann recognised the probability of facing cases as illustrated above. In this section of the sixth edition he has added a line, "and much less to give both together", and referred to the note of section 272, which is quoted** here, "some homoeopathists have made the experiment, in cases where they deemed one remedy homoeopathically suitable for one portion of the symptoms of a case of disease, and a second for another portion, of administering both remedies at the same or almost at the same time; but I earnestly deprecate such a hazardous experiment, which can never be necessary though it may sometimes seem to be of use." In this note although Hahnemann denounced the experiment of using more than one remedy at a time, yet he acknowledged the soundness of this by saying, "though it may sometimes seem to be of use". R. E. Dudgeon's* note in this connection is relevant here. The note runs as, "At the suggestion of Dr. Aegidi, Hahnemann was induced to try the effect in disease, especially chronic ones of mixing two highly diluted medicines and giving them in one dose. He was at first greatly pleased with the results obtained, and intended to recommend this plan in the fifth edition of the Organon, but was dissuaded from this by some of the most influential of his disciples; instead of doing so he merely alludes to the proposal, mildly denouncing it in the note of section 272".

With Dudgeon's note in view, one is logically led to believe that

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** The quotation is from 5th edition of the Organon.—Editor.

Hahnemann accepted the usefulness of the administration of more than one remedy at a time in cases where necessary.

SET ALGEBRAIC APPROACH TO USE OF MORE THAN ONE REMEDY AT A TIME

Let R_1 and R_2 be the remedies selected which are most suitable to cover the full symptom complex of the patient, and let $A_1^+(t)$ and $A_2^+(t)$ be the associative annihilation operators created by the administration of the remedies simultaneously, to the patient. $A_1^+(t)$ will act on the disease symptoms of $S(D)$ which are similar to the elements of the set of medicinal symptoms of $S(R_1)$ as recorded in materia medica. Similarly $A_2^+(t)$ will act on the symptoms of $S(D)$ which are similar to the elements of $S(R_2)$. Question may arise should not $A_1^+(t)$ effect $S(R_2)$ ∩ $S(D)$ or $A_2^+(t)$ effect $S(R_1)$ ∩ $S(D)$? Answer to these questions will be found in section 281. a part of which reads as, "Every patient is, especially in his disease *point* capable of being influenced *in an incredible degree* by medicinal agents corresponding by similarity of action;" $A_1^+(t)$ will not act on $S(R_2)$ ∩ $S(D)$ because it will not find any similarity of symptoms in $S(R_2)$ ∩ $S(D)$, but will act on $S(R_1)$ ∩ $S(D)$ because $A_1^+(t)$ will find similimum in $S(R_1)$ ∩ $S(D)$. By same argument $A_2^+(t)$ will only act on $S(R_2)$ ∩ $S(D)$, but not on $S(R_1)$ ∩ $S(D)$. Again we know from section 282 that the smallest possible dose of homoeopathic medicine bearing the greatest possible resemblance to the original disease, attack principally and almost solely the parts in the organism that are already affected, highly irritable, and rendered excessively susceptible to such a similar stimulus, and will alter the vital force that rules in them to a state of very similar artificial disease, somewhat greater in degree than the natural one was; this will substitute itself for natural (the original) disease, so that the living body now suffer from the artificial medicinal disease alone, which from its nature and owing to the minuteness of the dose, will soon be extinguished by the vital force that is striving to return to the normal state. This section is of vital importance to understand the effect of the simultaneous administration of R_1 and R_2 to the patient.

The simultaneous application of R_1 and R_2 to the patient is represented mathematically as

$$A_2^+(t) \; A_1^+(t) \; [S(D) \cap (S(R_1) \cup S(R_2))].$$

which expresses that the elements of the set of disease symptoms $S(D)$ are under the influence of the annihilation operators $A_1^+(t)$ and $A_2^+(t)$. Now by property of set algebra

$$S(D) \cap (S(R_1) \cup S(R_2)) = S(D) \cap S(R_1) \cup S(D) \cap S(R_2).$$

By virtue of the physical processes which have been expressed in sections 281 and 282, it is evident that during the simultaneous action of R_1 and R_2, $A_1^+(t)$ will act on $S(D)$ ∩ $S(R_1)$, and $A_2^+(t)$ on $S(D)$ ∩ $S(R_2)$.

So the simultaneous action of the remedies R_1 and R_2 on the patient may be mathematically represented as,

$$A_1^+(t) [S(D) \cap S(R_1)] \cup A_2^+(t) [S(D) \cap S(R_2)]$$

Since $S(R_1) \subseteq S(D)$, i.e. $S(R_1)$ is a subset of $S(D)$, one may write, $S(R_1) \cap S(D) = S^*(D)$, i.e. the set of disease symptoms not common to the elements of $S(R_1)$.

Since $S(D) = S(R_1) \cup S(R_2)$, for totality of symptoms one may write

$$S(R_2) \cap S(D) = S^*(R_2) = S(R_2),$$

and $S(R_1) \cap S(D) = S^*(R_1) = S(R_1)$.

Now, $A_1^+(t)$ acting on $[S(D) \cap S(R_1)]$, will eliminate the symptoms common in $S(R_1)$ and $S(D)$, and R_1 may produce some accessory symptoms $S_{R_1}(A)$ during the process of elimination of disease symptoms similar to $S(R_1)$

Therefore, one may write:

$$A_1^+(t) [S(D) \cap S(R_1)] = S(R_2) \cup S_{R_1}(A)$$

Similarly, $A_2^+(t) [S(D) \cap S(R_2)] = S(R_1) \cup S_{R_2}(A)$

By property of set algebra one may write:

$$A_1^+(t) [S(D) \cap S(R_1)] \cup A_2^+(t) [S(D) \cap S(R_2)]$$
$$= S(R_2) \cup S_{R_1}(A) \cup S(R_1) \cup S_{R_2}(A)$$

The physical interpretation of this equation is that as a result of simultaneous administration of the remedies R_1 and R_2 on the system (the patient), it has got rid of the elements of the set of disease symptoms $S(D)$, and is under the influence of the remedy symptoms of R_1 and R_2 and the accessory symptoms produced by them. This equation mathematically satisfies simultaneously provisions of sections 29 and 163 of the *Organon*. By provision of section 282 it is evident that the vital force itself will eliminate the remedy symptoms $S(R_1)$ and $S(R_2)$. The system in them left with the set of symptoms formed by $S_{R_1}(A) \cup S_{R_2}(A)$. If $S_{R_1}(A)$ and $S_{R_2}(A)$ belong to symptoms snffered by the patient in fhe past, then a third remedy R_3 will be necessary to match the symptoms of $S_{R_1}(A)$, $S_{R_2}(A)$, for its eradication. This state of affair corresponds to the provision of section 168. If R_3 cures the patient then one may write:

$$A_3^+(t) [S(R_3) \cap (S_{R_1}(A) \cup S_{R_2}(A))] = 0,$$

in the final state, which corresponds to cure state with complete eradication of the symptoms of the disease.

CONTROVERSY BETWEEN SINGLE REMEDY AND POLYPRESCRIPTION TREATMENT

"In no case it is requisite to administer more than one *single, simple, medicinal substance* at one time"—*said* Hahnemann in section 272, fifth edition. Again, "In no case under treatment is it necessary and *therefore not permissible* to administer to a patient more than *one, single, simple medicinal*

substance at one time. It is absolutely not allowed in Homoeopathy, the one true, simple and natural art of healing, to give the patient at one time two different medicinal substances",—section 273, sixth edition. Note of section 272 is a direct contradiction of these aphorisms. R. E. Dudgeon's note explains the reasons for the introduction of two contradictory ideas in the *Organon* by Hahnemann. A pertinent question may arise here, would a single remedy at a time always cure a case? It is clear that Hahnemann has been silent in describing the outcome of the administration of a single remedy at a time. He recognised that in a complicated disease when in addition to venereal chancre disease, sycotic or psoric disease are present, it is impossible to complete the cure with a single medicine. Here each appropriate homoeopathic (specific) remedy for one and the other must be employed alternately. Again the contents of section 163 and 168 recognise the fact that a single remedy does not always cure a patient. In the treatment of chronic diseases Hahnemann has accepted the use of interposed remedies for better results. In the note of section 246 fifth edition, he recommended the use of Hepar sulph. as interposed remedy with Sulphur in alternation. From the points discussed above, it is evident that Hahnemann recognised the fact that a single remedy does not always cure a patient, i.e. more than one remedy is required for a cure. If one relates sections 163 and 168 with the note of section 272, then it becomes evident that polyprescription or polypharmacy is logically valid and not unhomoeopathic. One might ask, could not polyprescription be harmful to a patient? Reference to sections 282 and 283 will indicate that it would not. If the patient for whom a polyprescription has been written be idiosyncratic[10] to one or more the selected remedies then the remedy or remedies will cure the disease symptoms and also develop its or their symptoms in the patient as if he is proving it/them. Reference to the note of section 141 will indicate that this instead of being harmful, it will be beneficial to him.

The author has been writing polyprescriptions for his patients since 1977, and his experience is that the results are very encouraging.

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REFERENCES

1. Hahnemann, S.: *Organon of Medicine*. English translation from the fifth edition by R. E. Dudgeon, Second Indian edition, p. 136. Roy Publishing House (1970).
2. *Ibid*. sections 157-160; 279.
3. *Ibid*. note, section 287.
4. *Ibid*. note, section 284.
5. *Ibid*. section 285.
6. *Ibid*. section 248; sixth edition.
7. *Ibid*. appendix, p. 232.
8. *Ibid*. appendix, p. 246.
9. *Ibid*. appendix, p. 249.
10. *Ibid* section 116-117.